THE APPROXIMATION IN REPLACING THE ACTUAL TEMPERATURE FIELD WITH A STATIONARY ONE IN THE SUBLIMATION PROCESS

A. Z. Volynets

Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 1, pp. 162–164, 1968

UDC 536.422.4

A formula is derived for estimating the error involved in replacing the actual temperature field with a stationary one in the process of sublimation drying. This estimate has been confirmed experimentally.

The difficulties of an analytic investigation of the sublimation drying process and the complexity of the physical model are due to the presence of a moving phase interface. Hence the importance of approximate methods, in particular, the replacement of the actual temperature field with a stationary one. We will estimate the error involved.

Consider a subliming body of arbitrary shape with internal heat sources. The volume density of heat release is a function of time and the coordinates. This formulation of the problem embraces all known methods of energy supply from high-frequency, when the sources are uniformly distributed over the thickness of the material, to conductive, when the sources can be assumed to be concentrated at the surface. We assume that the body is isotropic and homogeneous and that its physical parameters are constant; then the temperature must satisfy the equation

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t + \frac{e}{c\rho}. \qquad (1)$$

When the actual temperature field is replaced with a stationary one, it is assumed that the temperature in the body satisfies the equation

$$\nabla^2 t + \frac{e}{\lambda} = 0. \qquad (2)$$

Multiplying both sides of Eq. (1) by the volume element dV, integrating over the entire volume occupied by the body, and applying the Gauss theorem, we obtain

$$\int_0^{T_1}\int_{(S)} \lambda \operatorname{grad} t\,dS\,dt = \int_0^{T_1}\int_{(V)} c\rho \frac{\partial t}{\partial \tau}\,dV\,d\tau + \int_0^{T_1}\int_{(V)} e\,dV\,d\tau. \qquad (3)$$

Similarly, from Eq. (2)

$$\int_0^{T_2}\int_{(S)} \lambda \operatorname{grad} t\,dS\,dt = \int_0^{T_2}\int_{(V)} e\,dV\,dt. \qquad (4)$$

On the basis of the energy balance we represent Eq. (3) in the form

$$\lambda\, \overline{\operatorname{grad}_1} t T_1 \overline{S}_1 = \rho V r + k\rho c V \Delta t. \qquad (5)$$

The physical essence of the approximation consists in neglecting the specific heat of the substance investigated; therefore we represent Eq. (4) in the form

$$\lambda\, \overline{\operatorname{grad}}\, t\, T_2 \overline{S}_2 = \rho V r. \qquad (6)$$

We divide (5) by (6) and, noting that $\bar{S}_1 = \bar{S}_2$, we obtain

$$\frac{\overline{\operatorname{grad}_1} t T_1}{\overline{\operatorname{grad}_2} t T_2} = 1 \pm \frac{kc\Delta t}{r}. \qquad (7)$$

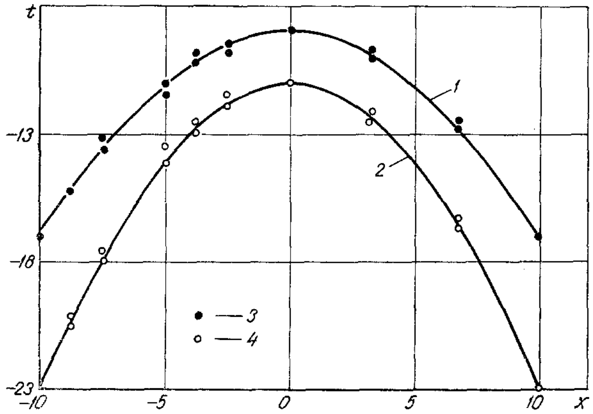
Obviously, the last term in (7) determines the maximum possible error for the total time of the process, when the actual temperature gradient at the surface is replaced with the gradient corresponding to a stationary distribution, or the error in determining the temperature gradient at the surface, when the actual time of the process is replaced with the time corresponding to the stationary process.

In ordinary sublimation drying processes, when the difference between the maximum and minimum ice temperatures is not more than 40° and the value of k does not exceed 1, the error in question does not exceed 3%.

Assuming that the deviation of the temperature gradient corresponding to the stationary process from the temperature gradient corresponding to the actual process is at a maximum at the surface, we can conclude that under certain conditions it is legitimate to replace the actual temperature field with a stationary one in investigating the sublimation drying process.

As a check, we measured the temperature in ice subliming in a high-frequency electric field. The temperature was measured with 12 specially designed thermocouples which excluded self-heating in the electric field and the usual distortion of the electric field in the specimen. The figure presents the results for a sheet of ice 20 mm thick at two different vapor pressures five minutes after switching on the high-frequency generator together with the temperature curves calculated on the assumption of a stationary temperature distribution, i.e., $t = -bx^2 + d$. Clearly, there is good agreement.

To obtain a more objective estimate of the degree of conformity of the experimental and calculated data,



Values of the temperatures t (°C) over the thickness x (mm) of a sheet of ice in the presence of sublimation in a high-frequency electric field: 1, 2) calculation; 3, 4) experimental values (1, 3—pressure 137 n/m^2; 2, 4—77 N/m^2).

by the method of least squares among the curves of the type $t = -bx^n + d$ we found the curve with parameters b and n approximating the experimental values best. At a confidence level of 95% we computed confidence intervals for the values of n, which for curves 1 and 2 in the figure are equal to 1.96 ± 0.042 and 2.03 ± ± 0.038, respectively. Thus, in both cases the confidence intervals include the theoretical value n = 2. Hence, at the assumed confidence level, the experimental data confirm the possibility of replacing the actual temperature field with a stationary one in the sublimation process.

NOTATION

t is the temperature; Δt is the difference between the maximum and minimum temperatures of the subliming material during the process; τ is the time; T is the total time of the process; c is the specific heat; ρ is the density; λ is the thermal conductivity; a is the thermal diffusivity; V is the volume; e is the volume density of heat release; k is the coefficient depending on the initial temperature distribution; r is the heat of phase transition; S is the surface; $\bar{S}$ is the mean surface over the entire duration of the process; grad t is the temperature gradient at the surface; $\overline{\text{grad}}$ t is the mean surface gradient; x is the distance from the center of the specimen. Subscript 1 denotes quantities corresponding to the actual and subscript 2 denotes quantities corresponding to the stationary process.

26 October 1967

Moscow Institute of Chemical
Engineering, Moscow